FURTHER EXTENSION OF THE MODIFIED RESIDUE CALCULUS TECHNIQUE

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Abstract

This paper extends the modified residue calculus technique to problems which previously have only been solved by the generalized scattering matrix technique in whole or part. In particular, the trifurcated waveguide solution is given.

Introduction

It is well known that the solution of a bifurcated waveguide can be obtained either by the Wiener-Hopf method or the residue calculus technique.[1] Pace and Mittra[2] originally used these known solutions in conjunction with the generalized scattering matrix technique to arrive at the solution of such composite problems as the E or H plane step. Later, Van Blaricum and Mittra[3] solved these same problems with the modified residue calculus technique using the idea of shifting known zeroes of the meromorphic function being constructed. Since the shift was known asymptotically the convergence was enhanced greatly over standard techniques. Recently, Royer and Mittra[4] examined a problem where the asymptotic shift of the zeroes could not be found. They then used the infinite form of the Lagrangian interpolating polynomial. The coefficients of the expansion could then be found asymptotically, restoring the enhanced convergence of the solution.

This paper studies a canonical problem of a bifurcated waveguide with infinitely many known modes incident from all guides. The solution of this problem can be expressed advantageously using the infinite form of the Lagrangian interpolating polynomial. This solution can be used to solve composite problems previously solved using the generalized scattering matrix technique. As an example, the E-plane trifurcated waveguide junction is solved. One interesting result is the asymptotic form of mode coefficients whose field must satisfy more than one edge condition.

Theory

Consider the TM solution of the bifurcated waveguide in figure 1. The fields are derivable from $\phi = H_y$ and the fields in each region are given by

$$\phi_A = \sum_{n=0}^{\infty}\left[A_n^{(o)} e^{\gamma_{na} z} + A_n e^{-\gamma_{na} z}\right]\cos\frac{n\pi}{a}(x-x_o)$$

$$\phi_B = \sum_{n=o}^{\infty}\left[B_n^{(o)} e^{-\gamma_{nb} z} + B_n e^{\gamma_{nb} z}\right]\cos\frac{n\pi}{b}(x-x_o)$$

$$\phi_C = \sum_{n=o}^{\infty}\left[C_n^{(o)} e^{-\gamma_{nc} z} + C_n e^{\gamma_{nc} z}\right]\cos\frac{n\pi}{c}(x-x_1)$$

where a superscript (o) indicates an incident field. Upon matching the fields at the junction and performing the usual procedures one can arrive at two infinite equations for the mode coefficient, A_n. For n = o we find immediately that

$$A_o = c/a\, C_o^{(o)} + b/a\, B_o^{(o)}$$

The solution to the problem is found by considering the following integrals

$$\frac{1}{2\pi j}\oint \frac{T(\omega)\,d\omega}{\omega\mp\gamma_{mc}},\quad \frac{(-1)^{m+1}}{2\pi j}\oint \frac{T(\omega)\,d\omega}{\omega\mp\gamma_{mb}}$$

where $T(\omega)$ has simple poles at γ_{na}, $-\gamma_{na}$, n=1,2, ... From the edge condition $T(\omega) = 0\ (\omega^{-2})$, $|\omega|\to\infty$ and hence the integrals are equal to zero. Then using the properties of $T(\omega)$ and comparing with the original equations for A_n we find the following properties of $T(\omega)$.

(i) $T(jk_o) = 2jk_o c e^{-jk_o Z_o}(A_o - C_o^{(o)})$

(ii) $T(\gamma_{mc}) = -C_m^{(o)}\gamma_{mc}\, c e^{-\gamma_{mc} Z_o}$

(iii) $T(\gamma_{mb}) = B_m^{(o)}\gamma_{mb}\, b e^{-\gamma_{mb} Z_o}(-1)^m$

(iv) $\mathrm{Res}[T,-\gamma_{ma}] = -A_m^{(o)}\frac{m\pi}{a}\sin\frac{m\pi b}{a} e^{\gamma_{ma} Z_o}$

(v) $\mathrm{Res}[T,\gamma_{ma}] = -A_n \frac{m\pi}{a}\sin\frac{m\pi b}{a} e^{-\gamma_{ma} Z_o}$

(vi) $T(-\gamma_{mc}) = C_m\gamma_{mc}\, c e^{-\gamma_{mc} Z_o}$

(vii) $T(-jk_o) = [C_o - A_o^{(o)}]2jk_o\, c e^{jk_o Z_o}$

(viii) $T(-jk_o) = -2jk_o\, b e^{jk_o Z_o}[B_o - A_o^{(o)}]$

(ix) $(-1)^{m+1}\, T(-\gamma_{mb}) = \gamma_{mb}\, bB_m\, e^{\gamma_{mb} Z_o}$

where m = 1,2, ... From these properties we can construct $T(\omega)$ as follows:

$$T(\omega) = g_E(\omega)\,\frac{\Pi(\omega,\gamma_b)\,\Pi(\omega,\gamma_c)}{\Pi(\omega,\gamma_a)}\,P(\omega) = F(\omega)P(\omega)$$

where

$$P(\omega) = K_o - (\omega - jk_o)\left[\sum_{n=1}^{\infty}\frac{g_n^{(b)}}{\omega-\gamma_{nb}} + \sum_{n=1}^{\infty}\frac{g_n^{(c)}}{\omega-\gamma_{nc}} + \sum_{n=1}^{\infty}\frac{g_n^{(a)}}{\omega+\gamma_{na}}\right]$$

and

$$g_E(\omega) = \exp\{-\omega/\pi[b\,\ell n\, b/a + c\,\ell n\, c/a]\}$$

where

$g_E(\omega)$ is determined from the edge condition; and where K_o, $g_n^{(b)}$, $g_n^{(c)}$ and $g_n^{(a)}$ are related to the incident fields via (i)-(iv). This represents the complete solution to the canonical problem.

The trifurcated waveguide can be solved by recessing the junction as shown in figure 2. This is only an artificial recession used as an aid in formulation of the problem. Actually, in computations $\Delta \equiv 0$. From the figure we recognize two distinct junctions. This leads us to construct two meromorphic functions. Let us identify a function $T_1(\omega)$ with the junction at $Z = 0$.

$$T_1(\omega) = F_1(\omega)\left[K_o^{(1)} - (\omega - jk_o)\sum_{n=1}^{\infty}\frac{g_n^{(1)}}{\omega+\gamma_{nc}}\right]$$

Similarly, let us identify a function $T_2(\omega)$ with the junction at $Z = \Delta$.

$$T_2(\omega) = F_2(\omega)\left[K_o^{(2)} - (\omega - jk_o)\sum_{n=1}^{\infty}\frac{g_n^{(2)}}{\omega-\gamma_{nc}}\right]$$

where $F_1(\omega)$, $F_2(\omega)$, $K_o^{(1)}$ and $K_o^{(2)}$ are known and are similar to $F(\omega)$ and K_o of the canonical problem with only a change of geometrical factors for each junction necessary. TEM mode incidence has been assumed. Extension to higher order mode incidence is straightforward. The functions $T_1(\omega)$ and $T_2(\omega)$ are just special cases of the canonical problem. The first function can be constructed by thinking of the unknown scattered modes from $Z = \Delta$ as the incident field of the junction. This corresponds to $g_n^{(c)} = g_n^{(b)} \equiv 0$ in the canonical solution. We then say $g_n^{(a)} = g_n^{(1)}$, recognizing that now $g_n^{(1)}$ represents a perturbation to $T_1(\omega)$ due to the coupling between the junctions. The second function is constructed similarly except now $g_n^{(b)} = g_n^{(a)} \equiv 0$, and $g_n^{(c)} = g_n^{(2)}$. Two infinite matrix equations can be derived by insisting that the two functions give consistent results for the modal coefficients in the coupling region. This results in the equations

$$\mathrm{Res}[T_1, \gamma_{nc}] = -\frac{n\pi}{c}\sin\frac{n\pi b_1}{c}\,[K_n^{(2)}]^{-1} g_n^{(2)}$$

$$T_2(-\gamma_{nc}) = \gamma_{nc}\, c\, [K_n^{(1)}]^{-1}\, g_n^{(1)}$$

where

$$g_n^{(1)} = K_n^{(1)}\, C_n^-$$

$$g_n^{(2)} = K_n^{(2)}\, C_n^+$$

$K_n^{(1)}$ is given by (iv), and $K_n^{(2)}$ is given by (ii) with appropriate changes of geometrical factors. The asymptotic behavior of $g_n^{(1)}$ and $g_n^{(2)}$ can be determined for the case $\Delta \equiv 0$ from these last equations to be

$$g_n^{(1)},\ g_n^{(2)} = 0\left(n^{-1}\sin\frac{n\pi b_1}{c}\right)$$

This choice is important since it allows the satisfaction of the edge condition at both edges. This allows truncation of the equations for enhanced convergence.

Numerical Results

Data was computed for the geometries in Pace and Mittra[5] and agreement was found in all cases (with exception of a slight error in Pace). No sensitivity to the ratio of the number of perturbation coefficients carried was found. Table 1 illustrates the convergence of the TEM current reflection coefficient of the region b_o for the parameters: $k_o b_o = 1.27046$, $k_o b_1 \cong 0.41417$, and $k_o b_2 = 0.20033$.

TABLE 1

N	|R|	Arg R
1	0.3230	131.1°
2	0.3244	132.0°
3	0.3243	132.0°
4	0.3241	131.8°
6	0.3243	131.9°
8	0.3243	131.9°

where N is the number of perturbation coefficients associated with each function (excluding the asymptotic coefficients). Pace gives a value of $R = 0.324\exp(j\,132.5)$ for this case. It should be noted that an independent check of the programming can be obtained by switching the dimensions, b_o and b_2. This result also converged to the above reflection coefficient.

Conclusions

This technique allows the solution of problems which previously have been solved using the generalized scattering matrix technique. The convergence is enhanced over the generalized scattering matrix technique as a result of the solution explicitly satisfying the edge condition at both edges.

This technique has also been extended to junctions involving more than three waveguides as well as junctions with various dielectric loading. The convergence of the numerical results are comparable to those shown for the trifurcated waveguide.

Currently, the results are being extended to open region type problems such as obtaining the coupling between two parallel plate waveguides above a homogeneous half space.

References

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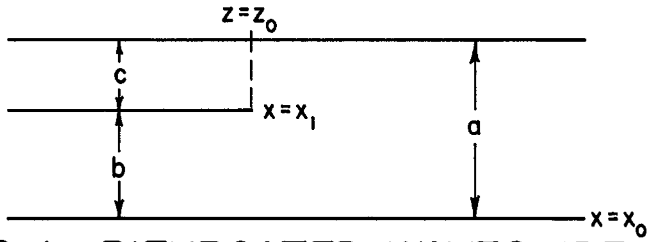

FIG. 1 BIFURCATED WAVEGUIDE

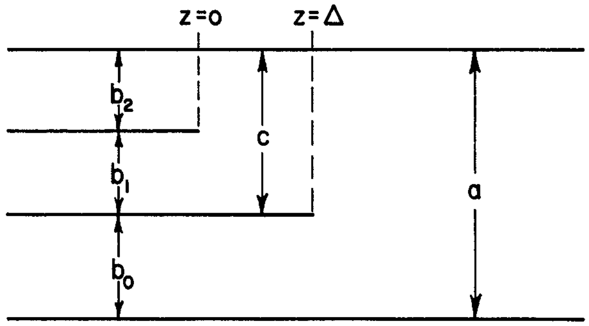

FIG. 2 TRIFURCATED WAVEGUIDE